Constitutive modeling of large-strain cyclic plasticity for anisotropic metals

1: Basic framework of modeling
2: Models of orthotropic anisotropy
3: Cyclic plasticity – Kinematic hardening model
4: Applications to sheet metal forming and some topics on material modeling

Fusahito Yoshida
Department of Mechanical Science and Engineering
Hiroshima University, JAPAN

Lecture 1: Contents

• Introduction:
  purpose of constitutive modeling,
• Stress and strain
• Yielding of isotropic solids
• Plastic potential and associated flow rule
• Isotropic/kinematic hardening models
• Isotropic hardening law
Material behavior under uniaxial tension

Necking occurs at a nominal stress peak, and it develops rapidly with increasing strain. The specimen fractures as a consequence of void nucleation, growth and coalescence.

Stress-strain curves of various metals
What material behaviors are our interests in plasticity modeling?

- Anisotropy (r-value, stress directionality)
- Cyclic plasticity (the Bauschinger effect, cyclic hardening, ratcheting, …, etc.)
- Damage (evolution of voids, …)
- Rate-dependent behavior (viscoplasticity, creep)
- Thermo-mechanical coupling
- … etc.

Modeling of Anisotropy and Hardening ($\sigma$-$\varepsilon$ responses) including the Bauschinger effect

![Anisotropy and Hardening Graphs](image)
Predictions of cracking and wrinkle

- Cracking
- Sheet thinning
- FE simulation

By PAM-STAMP 2G
(Yoshida-Uemori model)

DEFORMATION OF SOLIDS

- $F$: Deformation gradient
- $L$: Velocity gradient
- $D$: Rate of deformation (stretching) tensor
- $W$: Spin tensor
- $E$: Lagrangian strain tensor

\[
\begin{align*}
\frac{dx}{dt} &= F \cdot dX, \quad F_{ij} = \frac{\partial x_i}{\partial X_j} \\
\frac{dv}{dt} &= L \cdot dx, \quad L_{km} = \frac{\partial v_k}{\partial X_m} \\
D &= \frac{1}{2} \left( L + L^T \right), \quad W = \frac{1}{2} \left( L - L^T \right) \\
dE/dt &= F^T \cdot D \cdot F \approx D \\
D &= D^p + D^p
\end{align*}
\]
Stress (1)

Cauchy stress, principal stress

\[ \sigma = [\sigma_y] = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \]

\[ |\sigma_{ij} - \sigma^{(p)} \delta_{ij}| = 0, \sigma^{(p)} = \sigma_1, \sigma_2, \sigma_3 \]

Deviatoric stress and its Invariants

\[ s_{ij} = \sigma_{ij} - \sigma_m \delta_{ij} \]

\[ \sigma_m = \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) \]

= hydrostatic stress (or mean stress)

Stress (2)

Stress invariants

\[ J_1 = \sigma_{ii} = 3\sigma_m, \quad J_2 = \frac{1}{2} \sigma_{ij} \sigma_{ij}, \quad J_3 = \frac{1}{3} \sigma_{ij} \sigma_{jk} s_{ki} \]

\[ J'_1 = s_{ii} = 0, \quad J'_2 = \frac{1}{2} s_{ij} s_{ij}, \quad J'_3 = \frac{1}{3} s_{ij} s_{jk} s_{ki} \]

Jaumann rate (objective rate)

\[ \dot{\sigma} = \dot{\sigma} - W \sigma + \sigma W, \quad \dot{\sigma}_{ij} = \dot{\sigma}_{ij} - W_{im} \sigma_{mj} + \sigma_{im} W_{mj} \]
Initial Yielding of Isotropic Solids

Since the yielding is not affected by the hydrostatic stress component (i.e., incompressible), initial yielding of an isotropic solid is expressed by the function (yield function):

\[ f = f(\sigma_1, \sigma_2, \sigma_3) = f(J_1, J_2, J_3) = f(J_2', J_3') \]

For example,

**von Mises**

\[ f = \phi_M(s_{ij}) = \frac{3}{2}J_2' = \sigma_0^2 \]

**Drucker**

\[ f = \phi_D(s_{ij}) = 27\left(J_2^3 - \xi J_3^2\right) = \sigma_0^6 \]

---

Yield locus

Yield locus is a description of yield criterion in stress space.

- Carbon steel (S29C)
- Stainless steel (SUS304)
- Brass (BsBM1)
- Aluminum alloy (A2017)

Thin-walled tube in axial loads & internal/external pressure

Thin-walled tube in axial loads & torsion
Why is the yielding not affected by hydrostatic stress?

Physical background:
Plastic deformations in a crystal

Slip occurs most readily in specific directions (slip directions) on certain crystallographic planes (slip planes).

Schmid’s law: Slip of a crystal occurs when the resolved shear stress reaches its critical value, CRSS.

Resolved shear stress

\[ \tau^{(R)} = \sigma \cos \phi \cos \lambda \]

Schmid factor

Yield criterion for a crystal

\[ \tau^{(R)} = \frac{k}{k_{cr}} \] Critical resolved shear stress (CRSS)

Keywords: Slip system = Slip plane and slip direction
Resolved shear stress is not changed by the hydrostatic stress (pressure): $\tau_a = \tau_b$

At the atmosphere

\[ \sigma_1 = Y \]

Under hydrostatic pressure

\[ \sigma_1 = Y - p_e \]

Plastic potential & associated flow rule

Initial yield locus $F$

Loading

Neutral loading

Unloading

Subsequent yield locus $f$
\[ f = 0, \]
\[ \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \begin{cases} > 0 & \text{Loading} \\ = 0 & \text{Neutral loading} \\ < 0 & \text{Unloading} \end{cases} \]
Drucker’s postulate on stable stress-strain response

\[ d\sigma d\varepsilon = d\sigma \left( d\varepsilon^e + d\varepsilon^p \right) = E \left( d\varepsilon^e \right)^2 + d\sigma d\varepsilon^p \]

\[ d\sigma d\varepsilon^p \geq 0, \quad d\sigma_{ij} d\varepsilon_{ij}^p \geq 0 \]

(a) Stable                    (b) Unstable                (c) Multiaxial stress state

Stress cycle

\[ \int\int_{\sigma} (\sigma - \sigma^*) d\varepsilon = \int\int_{\sigma} (\sigma - \sigma^*) d\varepsilon^e + \int\int_{\sigma} (\sigma - \sigma^*) d\varepsilon^p \]

\[ = \int\int_{\sigma} (\sigma - \sigma^*) d\varepsilon^p \geq 0 \]

\[ \int\int_{\sigma} (\sigma_{ij} - \sigma_{ij}^*) d\varepsilon_{ij}^p \geq 0 \quad : \quad (\sigma_{ij} - \sigma_{ij}^*) d\varepsilon_{ij}^p \geq 0 \]
Principle of maximum plastic work

\[ (\sigma_{ij} - \sigma_{ij}^*) d\varepsilon_{ij}^p \geq 0 \quad \text{or} \quad (s_{ij} - s_{ij}^*) d\varepsilon_{ij}^p \geq 0 \]

- Convexity of yield locus
- Normality rule for plastic strain rate vector

\[ d\varepsilon_{ij}^p = \frac{\partial f}{\partial \sigma_{ij}} d\lambda \]

or

\[ d\varepsilon_{ij}^p = \frac{\partial f}{\partial s_{ij}} d\lambda \]

**Associated flow rule**

Yield locus of mild steel

Kuwabara
Hardening models

- Isotropic hardening
- Kinematic hardening
- Combined hardening

Workhardening

Isotropic hardening model

Initial yield

\[ F(\sigma) = \phi(\sigma) - Y^2 = 0 \quad \text{or} \quad F(s) = \phi(s) - Y^2 = 0 \]

Subsequent yield function

\[ f(\sigma) = \phi(\sigma) - [\sigma_o(\bar{\varepsilon})]^2 = 0 \]

or

\[ f(s) = \phi(s) - [\sigma_o(\bar{\varepsilon})]^2 = 0 \]

\[ f(s, \bar{\varepsilon}) = \frac{1}{2} s_y s_y - \frac{1}{3} \sigma_o(\bar{\varepsilon})^2 = 0 \]

\[ d\varepsilon_i^p = s_y d\lambda \]

\[ df = s_y ds_y - \frac{2}{3} H' \sigma_o d\bar{\varepsilon} = 0 \]

\[ H' = \frac{d\sigma_o}{d\bar{\varepsilon}} = \frac{d\bar{\varepsilon}}{d\bar{\varepsilon}} \]
Kinematic hardening model

Subsequent yield function

\[ f(\sigma_{ij}) = \phi(\sigma_{ij} - \alpha_y) - Y^2 = 0 \]

or

\[ f(s_{ij}) = \phi(s_{ij} - \alpha'_y) - Y^2 = 0 \]

Associated flow rule

\[ d\epsilon_{ij}^p = \frac{3d\bar{\epsilon}}{2Y}(s_{ij} - \alpha'_y) \]

Combined hardening model

Subsequent yield function

\[ f(\sigma_{ij}) = \phi(\sigma_{ij} - \alpha_y) - \sigma_o^2 = 0 \]

or

\[ f(s_{ij}) = \phi(s_{ij} - \alpha'_y) - \sigma_o^2 = 0 \]

Associated flow rule

\[ d\epsilon_{ij}^p = \frac{3d\bar{\epsilon}}{2\sigma_o}(s_{ij} - \alpha'_y) \]

Appropriate evolution equations for isotropic hardening and kinematic hardening is of vital importance.
Stress-Strain Response of a High Strength Steel Sheet of 590 MPa Grade

Isotropic hardening (IH) model

Stress-strain response of high strength steel sheet under reverse deformation characterized by the transient Bauschinger effect and permanent stress-offset.

Hardening law

= description of

- expansion of yield locus
- (isotropic hardening)

- movement of the center of yield locus (kinematic hardening: evolution of the back stress)
Isotropic Hardening Law by means of Effective Stress and Effective Plastic Strain

For initial yielding:
\[ F = \phi(s) - Y = \phi(s_{ij}) - Y = 0 \]

For the subsequent yielding:
\[ f = \phi(s) - (Y + R) = \phi(s_{ij}) - (Y + R) = 0 \]

Effective stress:
\[ \sigma = \phi(s) = \phi(s_{ij}) \]

For von Mises material:
\[ \sigma = 3J_2' = \frac{3}{2}s_{ij}s_{ij} \]

Effective Plastic Strain Increment \( d\bar{\varepsilon} \)

Work conjugate formulation:
\[ dw^p = \sigma_{ij} d\varepsilon_{ij}^p = s_{ij} d\varepsilon_{ij}^p = \sigma d\bar{\varepsilon} \]

When using von Mises effective stress:
\[ d\bar{\varepsilon} = \sqrt{\frac{2}{3}d\varepsilon_{ij}^p d\varepsilon_{ij}^p} \]

Effective plastic strain:
\[ \bar{\varepsilon} = \int d\bar{\varepsilon} = \int \dot{\varepsilon} dt \]
Isotropic Hardening Laws

\[ \bar{\sigma} = \bar{\sigma}(\bar{\varepsilon}) = Y + R(\bar{\varepsilon}) \]

For example,

- Linear hardening
  \[ \bar{\sigma} = Y + H'\bar{\varepsilon} \]
- Ludwik
  \[ \bar{\sigma} = Y + C\bar{\varepsilon}^n \]
- Swift
  \[ \bar{\sigma} = C\left(\varepsilon_0 + \bar{\varepsilon}\right)^n \]
- Voce
  \[ \bar{\sigma} = Y + R_{Sat} \left[1 - \exp(-\xi\bar{\varepsilon})\right] \]

Uniaxial tension stress-strain curves

- (a) Perfectly plastic solid
- (b) Linearly hardening plastic solid
- (c) Non-linearly hardening plastic solids

\[ \sigma = C(\varepsilon^p)^n \quad \text{Power-law hardening} \]
\[ \sigma = Y + C(\varepsilon^p)^n \quad \text{Ludwik} \]
\[ \sigma = C(\varepsilon_o + \varepsilon^p)^n \quad \text{Swift} \]
Constitutive modeling of large-strain cyclic plasticity for anisotropic metals

1: Basic framework of modeling
2: Models of orthotropic anisotropy
3: Cyclic plasticity – Kinematic hardening model
4: Applications to sheet metal forming and some topics on material modeling

Fusahito Yoshida
Department of Mechanical Science and Engineering
Hiroshima University, JAPAN

Lecture 2: Contents
1. What is the problem of anisotropy modeling?
   — Overview of some existing yield functions
     • Why Hill48 is still very popular? What’s the problem?
     • Polynomial type (Gotoh1977, Hu2007, Soare2008)
     • Linear transformation of stress tensor (Barlat’s Yld2000-2d)
     • Interpolation of biaxial stress states (Vegter2006)
2. A-user friendly 3D yield function
   • Yield function and its material parameter identification
   • Validation of the model
3. Application to FE simulations
   Hole-expansion and cylindrical deep drawing
4. Other types of modeling
Anisotropies of sheets (experimental)

R-value anisotropy & Flow stress directionality

Yield surfaces (equi-plastic work surfaces)

Bi-axial tension testing machine (Hiroshima University)

Biaxial experiment of sheet metals was first done by Shiratori & Ikegami, J. Mech & Phys Solids, 16 (1968), 373. Recently, Kuwabara is active in doing biaxial experiments (e.g., Int J Plasticity, 23 (2007), 385).

Yield surfaces of steel sheets

**IF steel**

\[ r_0 = 2.12, \quad r_{45} = 2.15, \quad r_{90} = 2.89 \]

\[ f = A_0 \sigma_x^1 + A_4 \sigma_y^4 + A_6 \sigma_y^6 + A_8 \sigma_y^8 \]

Hill48 quadratic yield criterion is acceptable

**HSS of 980 MPa-DP**

\[ r_0 = 0.73, \quad r_{45} = 0.91, \quad r_{90} = 0.81 \]

Hill48 quadratic yield criterion is acceptable
Material models for sheet metal forming simulation

Anisotropy + Bauschinger effect (Kinematic Hardening)
• F. Yoshida and T. Uemori: IJP 18 (2002), 61-686

Anisotropic yield function
Kinematic hardening

\[ f' = \phi (\sigma - \alpha) - Y = 0 \]
\[ D' = \frac{\partial f}{\partial \sigma} \lambda \]

Plastic potential theory

Note: Yield function should be a convex function of stress

A yield function is convex if its Hessian matrix:

\[ H_{ij} = \frac{\partial^2 \phi}{\partial \sigma_i \partial \sigma_j} \]

is positive semi-definite.
For yield function, what we need?

- **Accurate prediction of anisotropy**
  When using Hill48 yield function for hole-expansion simulation …

  ![Wrong prediction for the location of necking](image)

- **3D version of anisotropic yield function** (e.g., bottoming, ironning etc., NUMISHEET2011 Benchmark)

- Appropriate (robust and clear) scheme of material parameter identification

![Simulation](image)

Ogawa & Yoshida: NUMISHEET2011
Summary of some existing yield functions

1. **Hill48 quadratic function (Convex, 2D, 3D)**
   \[ \phi_H = A_1\sigma_x^2 - A_2\sigma_x\sigma_y + A_3\sigma_y^2 + 3A_4\tau_{xy}^2 = \sigma_0^2 \]
   * Material parameters \( A_1 \sim A_4 \) are identified using either \( r_0, r_{45} \) and \( r_{90} \) (Hill48-\( r \)) or \( \sigma_0, \sigma_{45}, \sigma_{90} \) and \( \sigma_b \) (Hill48-\( \sigma \)).

2. **Polynomial type (2D, 3D), e.g., Gotoh (1977)**
   \[ \phi_0 = B_1\sigma_x^3 - 2B_2\sigma_x^2\sigma_y + 3B_3\sigma_x^2\sigma_y^2 - 2B_4\sigma_x\sigma_y^3 - B_5\sigma_y^4 
   + 6(B_6\sigma_x^2 - B_7\sigma_x\sigma_y + B_8\sigma_y^2)\tau_{xy}^2 + 9B_9\tau_{xy}^4 = \sigma_0^4. \]
   * Material parameters \( B_1 \sim B_9 \) are identified using either \( r_0, r_{45} \) and \( r_{90}, \sigma_0, \sigma_{22.5}, \sigma_{45}, \sigma_{67.5}, \sigma_{90} \) and \( \sigma_b \).
   ** 3D version by Hu (IJP 23, 2007, pp. 620)
   + Convexity is not always guaranteed (Refer to S. Soare et al.: IJP 24(2008), pp. 915).

Why is Hill48 still very popular in industry?
What’s the problem?

- Easy material parameter identification from \( r_0, r_{45} \) and \( r_{90} \) solely.
- For weak anisotropy (e.g., some HSSs of \( r = 0.9 \sim 1.1 \) ), Hill48 describes it reasonably well. For an isotropic sheet, it is identical to von Mises.
- Convex 3D model.

\( r_0 = 2.12, r_{45} = 2.15, r_{90} = 2.89 \)
Predictions of r-values and flow stress directionality

- R-value anisotropy is well predicted by Hill48-r, where parameters are determined using \( r_0, r_{45}, \) and \( r_{90} \).
- Stress directionality is well predicted by Hill48-\( \sigma \), where parameters are determined using \( \sigma_0, \sigma_{45}, \sigma_{90}, \) and \( \sigma_b \).

3. Linear transformation of stress tensor (Barlat’s models)

\[
\text{Yld89} \quad \phi_{y} = a|K_1 + K_2|^{\mu} + a|K_1 - K_2|^{\mu} + (2 - a)|2K_1|^{\mu} = 2\sigma_0^{\mu}, \\
K_1 = \frac{1}{2}(\sigma_1 + h\sigma_0), \quad K_2 = \frac{1}{4}(\sigma_1 + h\sigma_0)^2 + (p\sigma_0)^2, \\
\text{Yld2000-2d} \quad \phi = \phi' + \phi'' = 2\sigma'' \\
\phi' = |X_1 - X_1'|^{\mu}, \quad \phi'' = |2X'_1 + X_1'|^{\mu} + |2X'_1 + X_1'|^{\mu} \\
X' = \mathbf{L}' : \sigma, \quad X'' = \mathbf{L}'' : \sigma \\
X_{12} = \frac{1}{2}(X'_1 + X'_2) \pm \frac{1}{2}\sqrt{(X'_1 - X'_2)^2 + 4X''^2} \\
\text{Material parameter are identified using } r_0, r_{45}, r_{90}, \sigma_0, \sigma_{45}, \sigma_{90}, \sigma_b. \\
\text{Extension to 3D is possible, but rather complicated. (Barlat et al.: IJP 23, 2007, pp. 1297)} \\
\text{Convexity is guaranteed}
What is the problem of Linear Transformation based 3D yield function using Principal Stresses?

\[
d\varepsilon_{ij}^p = \frac{\partial f}{\partial \sigma_{ij}} d\lambda = \frac{\partial f}{\partial X_k} \frac{\partial X_k}{\partial \sigma_{ij}} \frac{\partial s_{lm}}{\partial \sigma_{ij}} d\lambda
\]

Calculation is not so straightforward

\[
X_{1,2}^\prime = \frac{1}{2} \left( X_x^\prime + X_y^\prime \right) \pm \frac{1}{2} \sqrt{ \left( X_x^\prime - X_y^\prime \right)^2 + 4X_{xy}^2}
\]

2D is OK

4. Description of Biaxial Yield Locus by Bezier Interpolation (Vegter model)


\[
\sigma_{loc} = \bar{\sigma} + 2\mu (\bar{\sigma} - \bar{\sigma}) + \mu^2 (\bar{\sigma} + \bar{\sigma} - 2\bar{\sigma}), \quad 0 \leq \mu \leq 1
\]

* High flexibility of description of anisotropies.
** Convexity is guaranteed.
+ Very difficult to identify all the parameters.
++ 3D version impossible
A User-Friendly 3D Yield Function: New Proposition (Yoshida et al., NUMISHEET 2011)

2D version

\[ \phi = C_1 \sigma_x^2 - 3C_2 \sigma_y^2 \sigma_x + 6C_3 \sigma_x^4 \sigma_y - 7C_4 \sigma_x^3 \sigma_y^3 + 6C_5 \sigma_x^2 \sigma_y^4 - 3C_6 \sigma_x \sigma_y^6 + C_7 \sigma_y^6 \]

\[ + 9(C_7 \sigma_x^2 - 2C_8 \sigma_y^2 \sigma_x + 3C_9 \sigma_x^2 \sigma_y^2 - 2C_{10} \sigma_x \sigma_y^4 + C_{11} \sigma_y^4) \tau_{xy} \]

\[ + 27(C_7 \sigma_x^2 - C_{12} \sigma_x \sigma_y + C_{13} \sigma_y^2) \tau_{xy}^2 + 27C_{14} \tau_{xy}^6 \]

\[ = \sigma_0^6 \]

Normal-shear coupling terms

When \( C_1 = C_7 = \cdots = C_{16} = 1 \) this is identical to von Mises criterion.

16 material parameters

On material parameter identification

<table>
<thead>
<tr>
<th>Number of parameters</th>
<th>Flexibility &amp; accuracy</th>
<th>User-friendly &amp; Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td>Many</td>
<td>High</td>
<td>No</td>
</tr>
<tr>
<td>Few</td>
<td>Low</td>
<td>Yes</td>
</tr>
</tbody>
</table>

- A scheme of material parameter identification is very important for models with many parameters.
Convex Yield Function using Stress Invariants based on the Linear Transformation

\[ \phi^{(m)} = 27 \left( \tilde{J}_2^{(m)} - \xi \tilde{J}_3^{(m)} \right), \]

\[-27/8 \leq \xi \leq 9/4\]

\[ \tilde{J}_2^{(m)} = \frac{1}{2} \tilde{s}_{ij}^{(m)} \tilde{s}_{ij}^{(m)}, \quad \tilde{J}_2^{(m)} = \frac{1}{3} \tilde{s}_{ij}^{(m)} \tilde{s}_{jk}^{(m)} \tilde{s}_{ki}^{(m)} \]

\[ \phi = \frac{1}{n} \left( \phi^{(1)} + \phi^{(2)} + \ldots + \phi^{(n)} \right), \quad m = 1, 2, \ldots, n \]

Yoshida (2012) for highly flexible description of anisotropy

Description of Anisotropy based on Linear Transformation

\[ \tilde{S}^{(m)} = L^{(m)} \sigma \]

\[ L^{(m)} = \frac{1}{3} \begin{bmatrix}
 b_m + c_m & -c_m & -b_m & 0 & 0 & 0 \\
 -c_m & c_m + a_m & -a_m & 0 & 0 & 0 \\
 -b_m & -a_m & a_m + b_m & 0 & 0 & 0 \\
 0 & 0 & 0 & 3g_m & 0 & 0 \\
 0 & 0 & 0 & 0 & 3h_m & 0 \\
 0 & 0 & 0 & 0 & 0 & 3k_m
\end{bmatrix} \]
Two-step Material Parameter Identification

**Step 1:** \(a_m, b_m, c_m\) candidates

**Step 2:** \(a_m, b_m, c_m, g_m\) determination

- **Necessary reference points**
- **Others**

**Validation of the Model**

**Prediction of yield locus**

- Experimental data (590HSS) are from Hashimoto et al.: Tetsu-to-Hagane, 96(2010),27-33.
- \(r_0 = 0.43, r_{45} = 1.41, r_{90} = 0.61, \quad \bar{\sigma}_0 = 1.000, \bar{\sigma}_{45} = 0.936, \bar{\sigma}_{90} = 1.047, \bar{\sigma}_p = 1.000\)
Predictions of $r$-values and flow stress directionality

R-values: $r \alpha$

Flow stress directionality: $\sigma \alpha$

* Experimental data (590HSS) are from Hashimoto et al.: Tetsu to Hagane, 96(2010), 27-33.

Direction of plastic strain increment

* Experimental data (590HSS) are from Hashimoto et al.: Tetsu-to-Hagane, 96(2010), 27-33.
Prediction of yield locus

IF steel

* Experimental data (IF steel) are from Kitayama et al.: Tetsu-to-Hagane, 97(2011), 221-

$r_0 = 2.12, r_{45} = 2.15, r_{90} = 2.89, \bar{\sigma}_0 = 1.100, \bar{\sigma}_{45} = 0.988, \bar{\sigma}_{90} = 0.992, \bar{\sigma}_9 = 1.260$

IF steel ($r$-value)

When using $r_0$, $r_{45}$, and $r_{90}$ for parameter identification

A certain discrepancy
**IF steel \((r\text{-value})\)**

When using \(r_0\), \(r_{45}\) and \(r_{90} + r_{22.5}\) and \(r_{67.5}\) for parameter identification

**3D version of the model**

\[
\begin{align*}
\phi &= C_i (\sigma_z - \sigma_x)^4 - 3C_2 (\sigma_z - \sigma_x)^3 (\sigma_x - \sigma_y) + 6C_3 (\sigma_z - \sigma_x)^2 (\sigma_y - \sigma_z) \\
&\quad - 7C_4 (\sigma_z - \sigma_x)^2 (\sigma_x - \sigma_y) + 6C_4 (\sigma_z - \sigma_x) (\sigma_y - \sigma_z)^2 - 3C_5 (\sigma_z - \sigma_x) (\sigma_y - \sigma_z) + C_6 (\sigma_z - \sigma_x) \\
&\quad + 9(C_7 (\sigma_z - \sigma_x)^4 - 2C_8 (\sigma_z - \sigma_x)^3 (\sigma_x - \sigma_y) + 3C_9 (\sigma_z - \sigma_x)^2 (\sigma_y - \sigma_z) + C_10 (\sigma_z - \sigma_x)) \\
&\quad - 2C_11 (\sigma_z - \sigma_x) (\sigma_x - \sigma_y)^2 + C_12 (\sigma_x - \sigma_y)^4 (r_{xy}^2 + r_{xz}^2 + r_{yz}^2) \\
&\quad + 2\sigma_x (\sigma_x - \sigma_y)^2 - C_{14} (\sigma_x - \sigma_y) (\sigma_y - \sigma_z) + C_{15} (\sigma_x - \sigma_y)^3 (r_{xy}^2 + r_{xz}^2 + r_{yz}^2) \\
&\quad + 27C_6 (r_{xy}^2 + r_{xz}^2 + r_{yz}^2) = \sigma_0^4
\end{align*}
\]

* Different from Soare et al’s 3D expression (IJP 24 (2008), pp.915)

By assuming that anisotropies in terms of shear stress components \(\tau_{xy}\), \(\tau_{yz}\) and \(\tau_{zx}\) are equivalent, no additional parameters to 2D ones are needed for 3D version.

\(\tau_{yz}\) and \(\tau_{zx}\) are minor components in sheet metal forming
FE simulation of hole-expansion by LS-DYNA

Thickness strain distribution

* Experimental data (590HSS) are from Hashimoto et al.: Tetsu to Hagane, 6(2010), 27-33.

FE simulation of cylindrical deep drawing of 590R HSS (Earing)
Other Types of Modeling

1. Non-associated flow rule: Plastic potential $\phi$ is not the same as Yield function $F$

$$\sigma^0 = C - \frac{1}{\sqrt{3}} H \left[ \frac{\partial \phi}{\partial \sigma} \otimes \frac{\partial F}{\partial \sigma} + \frac{\partial F}{\partial \sigma} \right] : D$$

Non-symmetric matrix of stress-strain

e.g., Stoughton and Yoon, *Int J Plasticity* 25 (2009)

2. Distortion of Yield Locus

e.g., Shiratori et al. *JMPS* 27(1979), Barlat et al., *Int J Plasticity* 27 (2011)

Plastic potential

$$f_p = \phi(\sigma) - \sigma_p = 0;$$

$$D^p = \frac{\partial \phi}{\partial \sigma}$$

Yield locus

$$f_y = F(\sigma) - (Y + R) = 0;$$

$$f_y = \frac{\partial F}{\partial \sigma} : \sigma - R = 0$$
Constitutive modeling of large-strain cyclic plasticity for anisotropic metals

Fusahito Yoshida
Department of Mechanical Science and Engineering
Hiroshima University, JAPAN

Lecture 3: Contents

1. Introduction
2. Experimental observations of material behaviors of sheet metals in terms of anisotropy and cyclic plasticity.
3. Kinematic hardening laws: Cyclic Plasticity models
   - Linear KH (Prager), Mroz, Armstrong-Frederick (AF), Dafalias-Popov, Chaboche, Ohno-Wang, Teodosiu-HU, Yoshida-Uemori, etc.
4. Yoshida-Uemori model
5. Material parameter identification
INTRODUCTION
Press forming of high strength steel (HSS) and aluminum sheets is so difficult because of their nature of their large springback.
For accurate springback simulation, selection of a material model is very important.

Why are models of large-strain cyclic plasticity so important for springback analysis?

The accuracy of springback analysis depends on the predictions of stress levels at the final stage of stamping, and also at the springback, both which are directly related material’s Bauschinger effect and cyclic hardening characteristics.

Schematic illustration of stress-strain path during draw-bend and the subsequent springback.
Experimental observations of the Bauschinger effect & Cyclic Plasticity Characteristics

Schematic illustrations of in-plane cyclic tension-compression tests of sheet metals

* Other experimental techniques are by Wagoner (2004) and Kuwabara (2005).
Transient Bauschinger effect and the permanent stress offset in reverse deformation

Stress-strain response of high strength steel sheet under reverse deformation characterized by the transient Bauschinger effect and permanent stress offset.

Experimental observations of cyclic straining

Cyclic strain range dependency of cyclic hardening


Stress-strain responses of SPCC and SPCF (high-strength steel) under in-plane cyclic tension-compression (experimental data)
Effect of pre-strain on the subsequent cyclic behavior

Non-workhardening under small-strain cycling after large pre-strain

Stress-strain responses of SPCC under in-plane cyclic tension-compression with various pre-strains (experimental data)
The following material behavior should be modeled for sheet-metal forming simulation

**Bauschinger effect & Cyclic plasticity**
- Early re-yielding, transient Bauschinger effect and permanent stress offset in reverse deformation
- Workhardening stagnation
- Strain-range dependent cyclic workhardening

**Anisotropy** (r-value & flow stress directionality, biaxial flow stresses).


---

**Models of Large-Strain Cyclic Plasticity**  
*(Framework of modeling)*

With the assumption of small elastic strain and large plastic deformation, the rate of deformation $D$ is decomposed as:

\[
D = D^e + D^p
\]

\[
\dot{p} = \dot{\varepsilon} = \sqrt{(2/3)D^p : D^p}
\]

The constitutive equation of elasticity:

\[
\dot{\sigma} = \dot{\sigma} - \Omega \sigma + \sigma \Omega = C : D^p
\]

Objective rate of Cauchy stress  
Elasticity modulus

where

\[
W = W^p + \Omega
\]

Continuum spin  
Plastic spin  
Spin of substructures
Modeling of Cyclic Plasticity

Initial yield function

\[ f_0 = \phi(\sigma) - Y = 0 \]

Anisotropic yield function

\( \sigma \): Cauchy stress  \( Y \): Yield strength

Subsequent yield function and the associated flow rule

\[ f = \phi(\sigma - \alpha) - (Y + R) = 0, \]

\( D^p = \frac{\partial f}{\partial \sigma} \dot{\lambda} \)

Combined isotropic/kinematic hardening model

Several types Kinematic Hardening Laws
(Evolution equation of backstress)

- Linear KH (Prager 1949)
- A-F model (Armstrong-Frederick, 1966)
- Mroz (1967)
- Dafalias-Popov (1976)
- Chaboche (1979, 1983)

* are Large-Strain Cyclic Plasticity Model
Some of Kinematic Hardening Laws

- Linear KH (Prager 1949)
  \[ \dot{\alpha} = \frac{2}{3} C \dot{\varepsilon}^p \]

- A-F model (Armstrong-Frederick, 1966)
  \[ \dot{\alpha} = \frac{2}{3} C \dot{\varepsilon}^p - \gamma \dot{\alpha} \dot{\varepsilon} \]

- Chaboche (1979, 1983)
  \[ \alpha = \sum_{i=1}^{M} \alpha_i, \quad \dot{\alpha}_i = \frac{2}{3} C \dot{\varepsilon}^p - \gamma_i \dot{\alpha}_i \dot{\varepsilon} \]

  \[ \dot{\alpha} = \alpha - \beta = C \left[ \frac{\alpha}{Y} (\sigma - \alpha) - \frac{\alpha}{\sqrt{\alpha}} \right] \dot{\varepsilon} \]

Even by some complicated models, such as IH+AF-type NLK+LK model, cyclic hardening characteristics are so difficult to describe.
Constitutive Modeling of Large-strain Cyclic Plasticity for Anisotropic Sheets

**Yoshida-Uemori model**

Yoshida, F and Uemori, T:
*Int. J. Plasticity, 18* (2002), 661
*Int. J. Mechanical Sciences, 45* (2003), 1687

### Anisotropic Model of Large-Strain Cyclic Plasticity

**Initial yield function**

\[ f_0 = \phi(\sigma) - Y = 0 \]

**Anisotropic yield function**

\[ \sigma: \text{Cauchy stress} \quad Y: \text{Yield strength} \]

**Subsequent yield function and the associated flow rule**

\[ f = \phi(\sigma - \alpha) - Y = 0, \quad D^p = \frac{\partial f}{\partial \sigma} \]

**Bounding surface**

\[ F = \phi(\sigma - \beta) - (B + R) = 0 \]

**Backstress**

**Kinematic H**

**Isotropic H**

**Two surface model**
Kinematic/isotropic hadening (KH/IH) of 
**Yield surface and Bounding Surface**

For global workhardening, 
IH of \(B\)-surface 
\[
\dot{R} = nK^{1/n}R^{(n-1)/n} \dot{p}
\]
for Swift law

For permanent stress offset, 
KH of \(B\)-surface
\[
\beta' = k\left(\frac{2}{3}D^p : D^p - \beta \dot{p}\right)
\]

For the Bauschinger effect, 
KH of **Yield surface**
\[
\alpha_\ast = \alpha - \beta
\]
\[
\dot{p} = \sqrt{\frac{2}{3}}D^p : D^p, \bar{\sigma}_\ast = \phi(\alpha_\ast),
\]
\[a = B + R - Y\]

**Constitutive equation**

\[
\sigma = C - \sqrt{\frac{2}{3}H_{kin} \left[ \frac{\partial f}{\partial \sigma} \right] + \frac{\partial f}{\partial \sigma} : C \frac{\partial f}{\partial \sigma} } : D
\]

\[H_{kin} : \text{Rate of kinematic hardening}\]
\[
H_{kin} = \left[ \frac{Ca + kb}{Y} (\sigma - \alpha) - \left( C \sqrt{\frac{a}{\alpha_\ast}} + k \beta \right) \right] \frac{\partial f}{\partial \sigma}
\]
Description of Workhardening Stagnation by assuming non-IH of bounding surface

\[ \sigma_{\text{bound}}^{(0)} = B + R + \beta = B + (R_e + b)(1 - e^{-\epsilon_0}) \]

Explicit form!

Workhardening stagnation
Non-IH hardening of bounding surface

Schematic illustrations of the motion of: (a) the yield surface; and (b) the bounding surface under a uniaxial forward-reverse deformation.

Description of Workhardening Stagnation by non-IH stress surface model

When

\[ g_{\sigma} (\sigma', q', r) = 0 \quad \text{and} \quad \frac{\partial g_{\sigma}}{\partial \beta'} : \beta = 0 \]

\[ \dot{R} > 0 \quad [\text{hardening}] \]

Otherwise \( \dot{R} = 0 \) [non IH-hardening].

Kinematic motion and expansion of \( g_{\sigma} \)

\[ q' = \mu (\beta' - q') \]

\[ \mu = \frac{(1 - h) \Gamma}{r}, \Gamma = \frac{3(\beta' - q') : \beta}{2r} \]

When \( \dot{R} > 0, \dot{r} = h \Gamma \)
when \( \dot{R} = 0, \dot{r} = 0 \)

\[ 0 < h < 1 \]

Schematic illustration of the non-IH surface \( g_{\sigma} \), defined in the stress space, when (a) non IH; and (b) IH takes place.
Material Parameter Identification

- Automatic identification based on optimization technique
- M-Parameter identification tool: MatPara
The model involves seven parameters of cyclic plasticity + anisotropy parameters

- Yield strength: $Y$
- Kinematic hardening of yield surface: $C$
- Kinematic/isotropic hardening of bounding surface: $B, Rsat, k, b$
- Workhardening stagnation: $h$

These material parameters are systematically identified using experimental data of uniaxial tension and cyclic deformation.

Automatic parameter identification is possible by using optimization technique. (Material parameter identification tool: MatPara, CEM Inst. Co. Ltd.)

Material parameter identification by inverse approach using experimental data of uniaxial tension + cyclic plasticity

A set of material parameters: $x = [x_1, x_2, ..., Y, C, m, ...]$

Minimize the objective function:

$F(x) = \theta_1 F_1(x) + \theta_2 F_2(x)$

$F_1(x) = \sum_a \left( \frac{\sigma_{exp}^a - \sigma(x, \varepsilon_a)}{\sigma_{exp}^a} \right)^2$ for tension

$F_2(x) = \sum_a \left( \frac{\sigma_{exp}^a - \sigma(x, \varepsilon_a)}{\sigma_{exp}^a} \right)^2$ for cyclic
Material parameter identification by inverse approach using experimental data of cyclic bending

Performance of the model

- Cyclic plasticity behavior
- Non-proportional loading problem

F. Yoshida, M. Urabe and V. V. Toropov
Cyclic stress-strain responses under cyclic deformation calculated by the present model, together with the experimental results (Yoshida et al.) of high strength steel sheet (SPFC).

Strong Bauschinger effect appearing in 590 MPa HSS sheet

Y-U model can describe the Bauschinger effect, workhardening stagnation, strain range and pre-strain dependent cyclic hardening.

Cyclic stress-strain responses under cyclic deformation calculated by the present model, together with the experimental results (Yoshida et al.) of mild steel sheet (SPCC).
Description of yield plateau is possible by assuming a certain size of initial non-IH surface.

Stress-strain responses including yield-plateau on a mild steel sheet and its prediction by Yoshida-Uemori model.
HSS sheet (980 DP)  
(Y-U model + Hill 90 Yield function)

Aluminum sheet A5052  
(Y-U model + Barlat 2000 Yield function)
Stress-strain responses in strain path change

Equi-biaxial stretching

Uniaxial tension

\[ \varepsilon_1 = \varepsilon_2 \]

\[ \varepsilon_2 = -\frac{1}{2} \varepsilon_1 \]

Summary of Yoshida-Uemori model

- Accurate description of the Bauschinger effect and cyclic hardening characteristics including workhardening stagnation.
- Any types of anisotropic yield functions (e.g., Hill, Barlat, etc.) can be incorporated.
- Any types of uniaxial hardening law (e.g., Voce, Swift, etc.) can be incorporated.
- Limited number of material parameters (7+[1~2 for extended versions] parameters).
Constitutive modeling of large-strain cyclic plasticity for anisotropic metals

1: Basic framework of modeling
2: Models of orthotropic anisotropy
3: Cyclic plasticity – Kinematic hardening model
4: Applications to sheet metal forming and some topics on material modeling

Fusahito Yoshida
Department of Mechanical Science and Engineering
Hiroshima University, JAPAN

Lecture 4: Contents

1. Springback simulation

2. Springback compensation based on optimization technique

3. Some topics on material modeling
   - Modeling of yield point phenomena
   - Multi-scale modeling
   - Material database
Springback Simulation

- Hat-type draw bending
- S-rail forming
- Bumper beam
- B-pillar etc.

Accuracy of springback analysis strongly depends on material models.

CASE 1: Hat draw-bending

Experiment on 980HSS sheet

Experiment
Isotropic hardening

Yoshida–Uemori

Accurate description of the Bauschinger effect

(by LS-DYNA)
CASE 2: S-rail forming

Selection of a material model is of vital importance for accurate simulation of springback

CASE 3: Bumper beam

Selection of a material model is of vital importance for accurate simulation of springback

Yoshida-Uemori model
CASE 4:
Application of Yoshida model for mass-production parts

MAZDA-5(2005 model)'s B-pillar rein

Material: SPHN590R-DS t1.6 (Red)
SPCN780Y-N-E t1.8 (Green)
SPCN590R-N t1.4 (Blue)

1st forming (completed)

CASE 5: B-pillar (780+980 MPa HSS tailored blank)
Comparison between FE simulation (Pam-Stamp2G) and experimental results

Isotropic hardening
Yoshida-Uemori Kinematic hardening

Simulation error less than ±1.0mm
CASE 6: L-shaped beam

980HSS sheet

Simulation of wrinkles
by PAM-STAMP 2G: Yoshida-Uemori Model

No drawbead  2-mm drawbeads  4-mm drawbeads
Springback Compensation based on Optimization Technique

- Drawbeads for S-rail forming
- Tool shape design for bumper beam

Optimum Drawbead Setting for Springback Compensation in S-rail Forming

Twisting springback

Optimum drawbead
Effect of Drawbead on Springback

◆ Full drawbead setting

◆ Partial drawbead setting

Remove a part of draw bead line

Springback Control by Drawbead as a Problem of Optimization

Design variables $x = \text{Drawead heights}$

Where design variables $x = x_1, x_2, x_3, x_4$ are

- $0.0 \leq x_1 \leq 2.0$
- $0.0 \leq x_2 \leq 2.0$
- $-0.4 \leq x_3 \leq 0.4$
- $-0.4 \leq x_4 \leq 0.4$

Objective function to be minimized

$F(x) = \text{twisting angle}$

<table>
<thead>
<tr>
<th>No.</th>
<th>Drawbead height $H_i$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1</td>
<td>$H_1 = x_1$</td>
</tr>
<tr>
<td>No. 2</td>
<td>$H_2 = x_2$</td>
</tr>
<tr>
<td>No. 3</td>
<td>$H_3 = x_3 + H_1$</td>
</tr>
<tr>
<td>No. 4</td>
<td>$H_4 = x_4 + H_2$</td>
</tr>
<tr>
<td>No. 5</td>
<td>$H_5 = (H_1 + H_7) / 2$</td>
</tr>
<tr>
<td>No. 6</td>
<td>$H_6 = (H_2 + H_8) / 2$</td>
</tr>
<tr>
<td>No. 7</td>
<td>$H_7 = 2.0$</td>
</tr>
<tr>
<td>No. 8</td>
<td>$H_8 = 2.0$</td>
</tr>
</tbody>
</table>
**Result of FE simulation based Optimization**

- **No drawbead**
  
  ![Graph showing no drawbead](image)

  Tortional angle 3.5 degree

- **Optimum drawbeads**
  
  ![Graph showing optimum drawbeads](image)

  0.4 degree

Torsional springback is successfully suppressed by optimum drawbead setting

---

**Experimental Verification**

![Images of experimental setup and results](image)

- Blank holding
- Drawing
- Trimming

Optimum drawbead
**Experimental Verification**

- No draw bead
  - Torsional angle: 4.0 degree

- Optimum drawbeads
  - Torsional springback is successfully suppressed by optimum drawbead setting – **Verified!**

**Determination of optimum tool shapes for bumper beam**

- A. Cross-section opening springback
- B. Longitudinal springback

Springback compensation for A (cross section) and B (longitudinal) types were treated separately.
Die design as an optimization problem
(Cross section)

Minimize objective function $f(x)$
- Subject to
  $g_1(x) \leq C_1, g_2(x) \leq C_2, g_3(x) \leq C_3$

FE simulation Result after springback calculation

Objective function

Constraints

Die design as an optimization problem
(Longitudinal direction)

Objective function

Before springback

After springback

Target shape

Design variable

$X = I'$

8.1mm
Result of optimization
(Final shape of the beam after springback)

Some topics on material modeling

• Yield-point phenomena
• Multi-scale modeling
Rate-dependent Yield-Point Phenomena

Modeling and Simulations of Yield-Point Phenomena (Overview)

- **Metal physics**: Cottrell & Bibby (1949); Lomer (1952); …; Stein and Low (1966); …; Fujita & Miyazaki (1978); …; Neuhauser & Hampel (1993)

- **Constitutive modeling**: Johnston & Gilman (1959); Hahn (1962); Shioya & Shioiri (1976); Yoshida (IJP 16 (2000) 359)

- **Model of cyclic plasticity**


- **Polycrystal plasticity simulation**: Ghosh et al. (2004)
Framework of Constitutive Modeling

(1) Single crystal

\[ \dot{\gamma}^p = b \rho_m \nu \]
\[ \nu = \left( \frac{\tau_{\text{eff}}}{D_T} \right)^n \]
\[ \tau_{\text{eff}} = \tau - \tau_c \]

\[ \dot{\nu} = b \rho_m \left( \frac{\tau - \tau_c}{D_T} \right)^n \]

\[ b \]: Burgers Vector
\[ \rho_m \]: mobile dislocation density
\[ \tau_{\text{eff}} \]: effective resolved shear stress
\[ \tau_c \]: interaction stress acting on moving dislocations
\[ \nu \]: velocity of dislocations
\[ D_T \]: drag stress

Yield-point phenomena result from rapid dislocation multiplication and the stress-dependence of dislocation velocity.

---

Framework of Constitutive Modeling

(2) Polycrystals

\[ \dot{\varepsilon} = \frac{b \rho_m}{M} \left( \sigma - (Y + R) \right)^n \]
\[ \varepsilon^p = \frac{3(s-a)}{2\sigma} \dot{\varepsilon} \]
\[ \bar{\sigma} = \sqrt[3]{\frac{3}{2} (s-a) : (s-a)} \]

\[ s \]: stress deviator,
\[ \alpha \]: backstress deviator,
\[ R \]: isotropic hardening stress,
\[ Y \]: initial yield stress,
\[ M \]: Taylor factor

A Model of Yield Point Phenomena

Plastic deformation at Luders-band front:

\[ \dot{\varepsilon}_{LBF} = \frac{b \rho_m}{M} \left( \frac{\bar{\sigma} - Y}{D_{LBF}} \right)^n, \quad \bar{\sigma} = \frac{3}{2} s : s \]

Plastic deformation at workhardening region:

\[ \dot{\varepsilon}_{WH} = \frac{b \rho_m}{M} \left( \frac{\bar{\sigma} - (Y + R)}{D_{WH}} \right)^n, \quad \bar{\sigma} = \frac{3}{2} (s - \alpha) : (s - \alpha) \]


Rapid dislocation multiplication

\[ \rho_m = f \rho \]
\[ \rho = \rho_0 + C \varepsilon^a \]

(Hahn 1962; Kohda 1973; Hull & Bacon 1984)

\[ f = f_0 + (f_{asy} - f_0) \{ 1 - \exp(-\lambda \bar{\varepsilon}) \} \]

Initial value of mobile dislocation density is very small because of the Cottrell atmosphere.
Model of rapid dislocation multiplication

\[ \rho_m = f \rho \]
\[ \rho = \rho_0 + C \varepsilon^a \]

(Hahn 1962; Kohda 1973; Hull & Bacon 1984)

\[ f = f_0 + (f_{asy} - f_0) \{1 - \exp(-\lambda \varepsilon)\} \]

A sharp yield point and the subsequent abrupt yield drop is a consequence of rapid dislocation multiplication and strong stress dependency of dislocation velocity.

Very low mobile dislocation density because of Cottrell locking

Uniaxial tension

Experiment

Simulation

Elimination of Yield-Point by temper rolling

A model for $\beta$-Ti (Ti-20V-4Al-1Sn) at elevated temperature

\[
\dot{\varepsilon}^p = \frac{b_p m}{M} \left( \frac{\sigma - Y_0 - R_{\text{inh}}}{D} \right)^n \exp \left( -\frac{Q}{RT} \right)
\]

\[
\dot{R}_{\text{iso}} = B(Q_{\text{iso}} - R_{\text{iso}}) \dot{\varepsilon}^p - aR_{\text{iso}}
\]

Strain hardening
Dynamic recovery

X.T. Wang, F. Yoshida et al.: Mat Trans 50-9 (2009), pp.1576

Multi-scale modeling for prediction of macro elasto-plasticity behavior of materials

Macro modeling

Material parameters associated with micro structures

Continuum mechanics

Crystal plasticity

DD, MD

Dislocation motion, accumulation and D-structure formation

Models of obstacles (G-boundaries, precipitates, etc.)

FE forming simulation
Homogenization

Modeling for single crystal for each phase

Volume fraction of each phase and texture

Modeling for multi-phase & polycrystal materials

Continuum mechanics
Material tests & Parameter identification

Material Database

Cyclic plasticity

Material parameter identification

Automatic identification software

Yield function, material parameters

Database

Sheet metal forming simulation

Forming limit criteria & material parameters

SPCN780Y