Strain Gradient Plasticity with Microstructural Characterization for Impact Damage and Perforation

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Abstract:

The objective of this work is to present a micromechanically based constitutive model for the modeling of heterogeneous media that assesses a strong coupling between (visco) plasticity and (visco) damage for high velocity impact related problems while considering the discontinuities on the macroscale level. The essential aspects of interest here can all be examined within the context of: (1) finite deformation kinematics; (2) rapid time variations in temperature, strain, strain rate, and other field variables; (3) equation of state; (4) thermal and damage softening; (5) strong viscoplasticity and viscodamage coupling; (6) long range microstructural interactions through the use of the nonlocal continua; and (7) numerical stability through the use of regularization approaches (i.e., using viscosity and gradient localization limiters).

Constitutive Model:

The model is based on the nonlocal gradient plasticity and gradient damage theories. It includes the nonlocal von Mises yield criterion, the non-associated flow rules, isotropic and anisotropic strain hardening, strain rate hardening, softening due to adiabatic heating and anisotropic damage evolution, and finally a path dependent equation of state. The stress-strain rate relationship in the spatial and damaged configuration is given by

$$\overset{\nabla}{\boldsymbol{\tau}} = \mathbb{C} : \left(\boldsymbol{d} - \boldsymbol{d}^{vp} - \boldsymbol{d}^{vd} \right) - \boldsymbol{A} : \overset{\nabla}{\boldsymbol{\phi}} - \mathbb{C} : \alpha_t \dot{T} \mathbf{1} \tag{1}$$

where $\mathbb{C} = \widehat{\boldsymbol{M}}^{-1} : \overline{\mathbb{C}} : \widehat{\boldsymbol{M}}^{-1}$ and $\widehat{\boldsymbol{M}} = 2\left[\left(\mathbf{1} - \widehat{\boldsymbol{\phi}} \right) \otimes \mathbf{1} + \mathbf{1} \otimes \left(\mathbf{1} - \widehat{\boldsymbol{\phi}} \right) \right]^{-1}$. The notation, ∇ indicates co-rotational objective derivative, τ is the Kirchhoff stress tensor, d is the total rate of deformation, $\boldsymbol{d}^{vp}$ is the viscoplastic rate of deformation, $\boldsymbol{d}^{vd}$ is the viscodamage rate of deformation, $\overline{\mathbb{C}}$ and $\mathbb{C}$ are the fourth-order undamaged and damaged elasticity tensors respectively, α_t is the thermal expansion coefficient, $\dot{T}$ is the rate of absolute temperature, and $\mathbf{I}$ is the second-order identity tensor

The viscoplastic and viscodamage multipliers, $\dot{\lambda}^{vp}$ and $\dot{\lambda}^{vd}$, can be obtained in a nonlocal sense using the following generalized Kuhn-Tucker conditions for rate-dependent problems such that

$$\dot{\lambda}^{vp} \geq 0, \quad f \leq 0 \quad \Leftrightarrow \dot{\lambda}^{vp} f = 0 \quad \text{and} \quad \dot{\lambda}^{vd} \geq 0, \quad g \leq 0 \quad \Leftrightarrow \dot{\lambda}^{vd} g = 0 \tag{2}$$

The equation of state is a thermodynamic equation describing the state of matter under a given set of physical conditions. In this sense the thermodynamic pressure stress P for a shock compressed solid is given as follows:

$$P = \left(1 - \gamma\right) c_v T^{ig} \varepsilon^e \tag{3}$$

where $T^{ig} = T_r \exp\left[\left(\eta - \eta_r \right) / c_v \right] \left[1 + \varepsilon^e \right]^{(\gamma - 1)} \exp\left[\left(\gamma - 1 \right) \left(1 / \left(1 + \varepsilon^e \right) - 1 \right) \right]$

To establish the actual heat generation that occurs during the highly transient impact events of the thermo-mechanically coupled finite element, the development of a heat equation is imperative. However, because the whole impact process lasts a few hundred of a μs, the effect of heat conduction is negligible over the domain of the specimen and therefore, an adiabatic condition is assumed such that the increase in temperature is calculated by the following heat equation:

$$\rho_o c_p \dot{T} = \Upsilon \boldsymbol{\tau}' : \left(\boldsymbol{d}^{vp} + \boldsymbol{d}^{vd} \right) + J^e P(\boldsymbol{d}^e : \mathbf{1}) \tag{4}$$

Voyiadjis and Abu Al-Rub (2005) showed that ℓ is not a fixed parameter and depends on the mean grain size d. This is in line with the dependence of the mean free path distance L_S on the plastic strain p, the hardening exponent m, and the grain size d. One, therefore, requires an evolution equation for the material length scale that is consistent with the experimental trends, such that the following expression for the length scale parameter ℓ can be adopted:

$$\ell = \frac{\hbar D d}{D + d\, p^{1/m}} \tag{5}$$

Fig. 1 shows the element distortion in the target plate just after impact. The final cross section of a target plate perforated by a blunt projectile at an impact velocity close to the ballistic limit is shown in Fig. 2.

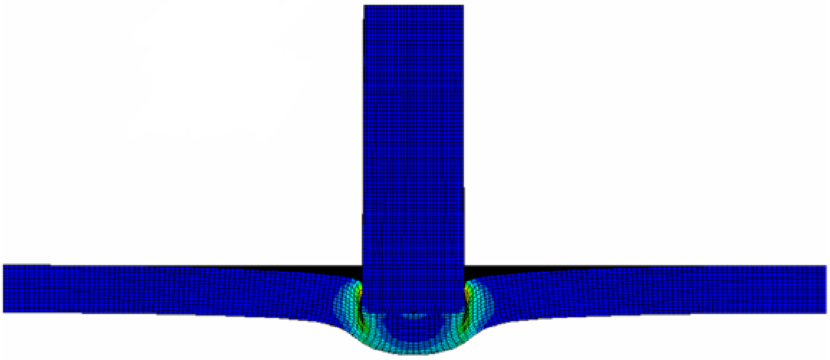

Fig. 1. Penetration of the target plate by a blunt projectile of initial impact velocity of $300\ m/s$ *using ALE meshing, plotted as contours of accumulated viscoinelastic strain and showing details of element meshes just after adaptive remeshing (Voyiadjis and Abu Al-Rub, 2006).*

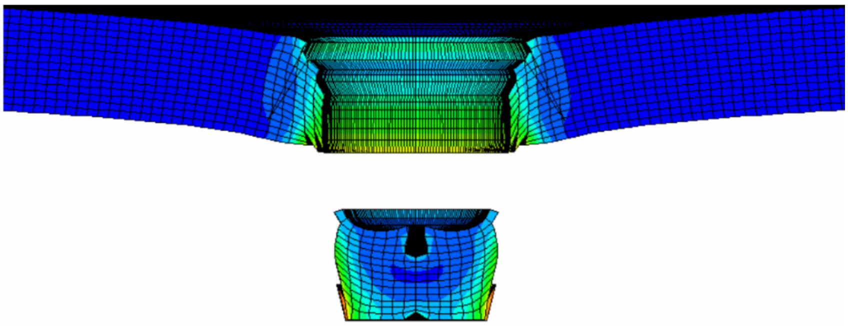

Fig. 2. Final cross section of the target plate perforated by a blunt projectile of initial impact velocity of $300\ m/s$ *using ALE meshing and plotted as contours of accumulated viscoinelastic strain. The green indicates an accumulated damage between* 0.25 *and* 0.30 *(Voyiadjis and Abu Al-Rub, 2006).*

References

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[2] Voyiadjis, G.Z., Abu Al-Rub, R.K., 2006. A finite strain plastic-damage model for high velocity impacts using combined viscosity and gradient localization limiters: Part II-numerical aspects and simulations. International Journal of Damage Mechanics 15, 335-373.